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MATHEMATICS

PROBABILITY, PERMUTATIONS AND COMINATIONS_SYNOPSIS

•	**The numbers of ways of dividing 'mn' different things into 'm' equal groups each containing 'n' elements** $=\frac{(mn)!}{m!(n!)^m}$
•	**The numer of ways of distributing 'mn' different things equally among 'm' persons** $=\frac{(mn)!}{(n!)^m}$
•	**The number of diagonal in an n sided polygon** $=n_{c_2}-n$
•	**For** $0\le r,s\le n,$ **if** $n_{c_r}=n_{c_s},$ **then either** $r=s$ **or** $r+s=n$**.**
•	$n_{c_r}+n_{c_{r-1}}=n+1_{C_r}$
•	**If p things are alike of one kind and 'q' things are alike of second kind and r things are alike of third kind, then the number of ways of selecting only number of things (one or more) out of these** $(p+q+r)$ **things is** $(p+1)(q+1)(r+1)-1.$
•	**The number of ways of selecting one or more things out of 'n' distinct things** $=2^n-1.$
•	**If** $p_1,p_2,.........p_k$ **are distinct primes and** $\alpha_1,\alpha_2,....\alpha_k$ **are the integers, then the number of the divisors of** $n=p_1^{\alpha_1},p_2^{\alpha_2}.....p_k^{\alpha_k}$ **is** $(\alpha_1+1)(\alpha_2+1).......(\alpha_k+1)$ **this includes '1' and 'n'**
•	**In the above, the numbers of proper divisors of n is** $(\alpha_1+1)(\alpha_2+1)....[\alpha_{k+1}]-2.$
•	**Exponent of a prime 'p' in n ! is the largest integer k such that** p^k **divides n !**

•	$E_p(n!)=\left[\frac{n}{p}\right]+\left[\frac{n}{p^2}\right]+\left[\frac{n}{p^3}\right]+......$ **where** $[\bullet]=G.I.F$
•	$n_{P_r}=n.^{n-1}P_{r-1}=n(n-1).^{n-2}P_{r-2}etc$
•	$n_{P_r}={}^{(n-1)}P_r+r.^{(n-1)}P_{(r-1)}$
•	**Sum of all r – digit numbers that can be formed using the given 'n' non-zero digits** $[1\le r\le n\le 9]$ **is** ${}^{(n-1)}P_{(r-1)}(sum\ \ of\ \ the\ given\ digits)\times 111.....1(rtimes)$
•	**In the above. If 'O' is one digit among n digit , the we get that the sum of the r-digit numbers that can be formed using the given n digits including 'O'** $=\left\{{}^{(n-1)}P_{(r-1)}\times sum\ of\ \ the\ given\ digits\ \times 111...1\ (r\ times)\right\}$ $-\left\{{}^{(n-2)}P_{(r-2)}\times sum\ of\ \ the\ given\ digits\times 111...1\ (r-1)\ times\right\}.$
•	$Let\ \ f:A\to B$ **be a function then** **a)The number of functions** $=n(B)^{n(A)}$ **b)The number of injections** $=\begin{cases}{}^{n(B)}P_{n(A)}, & if\ n(A)\le n(B)\\ O & if\ n(A)>n(B)\end{cases}$ **c)The number of surjection's** $=2^{n(A)}-2, \quad if\ n(B)=2$
•	**If there are n things in a row, a permutation of these n things such that none of then occupies its original position is called a derangement of n thigns.** **The number of derangements of n distinct things** $=n!\left(\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}+.......+(-1)^n\frac{1}{n!}\right)$
•	**Palindrome : A number or a word which reads same either from left to right or from right to left is called a palindrome, examples, 120021, ROTOR etc**

•	**The number of palindromes with r distinct letters that can be formed using the given n distinct letters is** **a)** $n^{r/2}$, **if r is even** **b)** $n^{\frac{r+1}{2}}$ **, if r is odd**
•	**The number of circular permutations of n distinct things taken all at a time** $n^{\frac{r+1}{2}} = (n-1)!$
•	**In case of the garlands of flowers, chains of beads etc. clock-wise and anti clock –wise arrangements will be treated as identical there fore number of circular permutations of n things** $= \frac{(n-1)!}{2}$.
•	**The number of linear permutations of 'n' things in which there are p like things of one kind, 'q' like things of 2nd kind, r like things of third kind and rest different is** $\frac{n!}{p!q!r!}$
•	**The number of different subsets of 'r' elements of a set containing 'n' elements** $= n_{c_r}$ **.**
•	$n_{C_r} = n_{C_{n-r}}$ **The numbers of ways of dividing** $m+n$ **things in to two group containing m things and n things is** $\frac{(m+n)!}{m!n!}$ **(m,n are distinct +ve integers)**
•	**If A and B are two events then** **(i)P(both A,B occurs) = P** $(A \cap B)$, **(ii) P (at least one of A,B occurs)** $= P(\bar{A} \cup \bar{B})$ **(iii) P (none of A,B occurs)** $= P(\bar{A} \cap \bar{B})$ **(iv) P (exactly one of A,B occurs) =** $P(A \cap \bar{B}) + P(B \cap \bar{A}) = P(A) + P(B) - 2P(A \cap B)$ **(or)** $P(A \cup B) - P(A \cap B)$

•	**If A,B and C are three events then** **(i)P(at least one of A,B,C occurs)** $=P\left(A\cup B\cup C\right)$ **(ii) P (at two A,B,C occur)** $=P\left(A\cap B\cap \bar{C}\right)+P\left(A\cap \bar{B}\cap C\right)+$ $P\left(\bar{A}\cap B\cap C\right)+P\left(A\cap B\cap C\right)$ $=P\left(A\cap B\right)+P\left(B\cap C\right)+P\left(C\cap A\right)-2P\left(A\cap B\cap C\right)$ **(iii) P (exacty two of A,B,C occurs** $=P\left(A\cap B\cap \bar{C}\right)+P\left(A\cap \bar{B}\cap C\right)+P\left(\bar{A}\cap B\cap C\right)$ $=P\left(A\cap B\right)+P\left(B\cap C\right)+P\left(C\cap A\right)-3P\left(A\cap B\cap C\right)$ **(iv) P (exactly one of A,B,C occurs)** $=P\left(A\cap \bar{B}\cap \bar{C}\right)+P\left(\bar{A}\cap B\cap \bar{C}\right)+P\left(\bar{A}\cap \bar{B}\cap C\right)$ $=P\left(A\right)+P\left(B\right)+P\left(C\right)-2P\left(A\cap B\right)$ $-2P\left(B\cap C\right)-2P\left(C\cap A\right)+3P\left(A\cap B\cap C\right)$
•	**When n fair dice are rolled once the number of favourable cases to get the sum r is coefficient of** x^r **in the multinomial expansion of** $\left(x^1+x^2+x^3+x^4+x^5+x^6\right)^n$
•	**Two Person game : (infinite G.P mode 1) If p and q are the probabilities of success and failure of a game in which A and B play and if A starts the game then,**

•	**Dearrangement Problems** **If n letters corresponding to n envelopes with address on them are placed in the envelopes one in each, then** **i)probability that all the letters are placed in right envelopes** $=\frac{1}{n!}$ **ii) probability that all letters are not in right envelopes (or at least one letter is placed in wrongly addressed envelope)** $=1-\frac{1}{n!}$ **iii) probability that all letters go into wrongly addressed envelopes** $=\left[\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-.......+\frac{(-1)^n}{n!}\right]$ **(iv) Probability that exactly r letters are in right** **Envelops** $=\frac{1}{r!}\left(\frac{1}{2!}-\frac{3}{3!}+\frac{1}{4!}-.......+(-1)^{n-r}\frac{1}{(n-r)!}\right)$
•	**Using the vertices of a polygon having n sides a triangles is constructed at random. The probability that the triangle so formed is such that no side of the polygon is side of the triangle is** $\frac{(n-4)(n-5)}{(n-1)(n-2)}$
•	**Out of n persons at a round table, three persons are selected at random then the probability that no two of them are consecutive is** $\frac{(n-4)(n-5)}{(n-1)(n-2)}$.

PROBABILITY, PERMUTATIONS AND COMINATIONS_ASSIGNMENT

1. A fair coin whose faces are marked with 3 and 4 instead head and tail, is tossed 4 times. The odds against the sum of the numbers thrown being less than 15, is

1) 1:15 **2)** 5:11 **3)** 1:3 **4)** 3:13

2. Let A be the event that an year chosen at random, is a leap year and B be the event that an year chosen at random to have 52 sundays , then which of the following statement is wrong

1) P(B) =23/28 **2)** P(A/B) =5/23 **3)** $P(A/B^c) = 2/5$ **4)** $P(A^c/B) = 2/5$

3. An arbitrary cube has four blank faces, one face marked 2 and another marked 3. Then the probability of obtaining a total of exactly 12 in 5 throws is

1) $\frac{5}{1296}$ **2)** $\frac{5}{1944}$ **3)** $\frac{5}{2592}$ **4)** $\frac{11}{1294}$

4. An unbiased die is rolled 4 times. Out of 4 face values obtained, the probability that the minimum face value is not less than 2 and the maximum face value is not greater than 5 is

1) $\frac{16}{81}$ **2)** $\frac{1}{81}$ **3)** $\frac{80}{81}$ **4)** $\frac{65}{81}$

5. If 100 boys are arranged at random along a circle then the odds against to arrange two specified boys of those 100 come together is

1) 2:97 **2)** 97:2 **3)** 2:98 **4)** 98:2

6. If 10 identical coins are distributed among 4 children at random. The probability of distributing so that each child gets atleast one coin is

1) $\frac{12}{143}$ **2)** $\frac{42}{143}$ **3)** $\frac{17}{143}$ **4)** $\frac{101}{143}$

7. If a is an integer and $a \in (-5, 30]$ then the probability that the graph of the function $y = x^2 + 2(a+4)x - 5a + 64$ is strictly above the x-axis is

1) $\frac{1}{5}$ **2)** $\frac{8}{25}$ **3)** $\frac{8}{35}$ **4)** $\frac{27}{35}$

8. If A and B are events of a random experiment such that $P(A \cup B) = \frac{4}{5}, P(\bar{A} \cup \bar{B}) = \frac{7}{10}$ and $P(B) = \frac{2}{5}$, then $P(A) =$

1) $\frac{9}{10}$ **2)** $\frac{8}{10}$ **3)** $\frac{7}{10}$ **4)** $\frac{3}{5}$

9. A and B are two independent events. The probability that both A and B occur is 1/6 and the probability that neither of them occur is $\frac{1}{3}$. Then the probability of occurrence of A is

1) $\frac{1}{2}$ **2)** $\frac{1}{4}$ **3)** $\frac{1}{6}$ **4)** $\frac{1}{8}$

10. The odds in favour of A winning a game of chess against B are 5:2. If 3 games are played then the odds in favour of A winning atleast one game are

1) 335 : 8 **2)** 8 : 335 **3)** 335 343 **4)** none

11. India plays two matches each with West Indies and Australia. In any match the probabilities of and 0.50 respectively. Assuming that the outcomes are independent, the probability of getting atleast 7 points is

1) 0.8750 **2)** 0.0875 **3)** 0.0625 **4)** 0.0250

12. Suppose x is a binomial distribution with parameters $n=100$ and $p=\frac{1}{2}$ then $P(x=r)$ is maximum when r =

1) 50 **2)** 32 **3)** 33 **4)** 67

13. Six coins are tossed 6400 times. The probability of getting 6 heads x times using poison distribution is

1) $6400e^{-x}$ **2)** $\frac{6400e^{-x}}{\angle x}$ **3)** $\frac{e^{-100}100^{x}}{x!}$ **4)** e^{-100}

14. If $\alpha = {}^{m}C_2$, then ${}^{\alpha}C_2$ is equal to

1) ${}^{m+1}C_4$ **2)** ${}^{m-1}C_4$ **3)** $3\,{}^{m+2}C_4$ **4)** $3\,{}^{m+1}C_4$

15. The number of possible outcomes in a throw of n ordinary dice in which at least one of the dice shows an odd number is

1) $6^n - 1$ **2)** $3^n - 1$ **3)** $6^n - 3^n$ **4)** none of these

16. The number of words of four letters containing equal number of vowels and consonants, where repetition is allowed , is

1) 105^2 **2)** 210×243 **3)** 105×243 **4)** 150×21^2

17. The number of ways in which ten candidates $A_1, A_2, ..., A_{10}$ can be ranked such that A_1 is always above A_{10} is

1) 5 ! **2)** 2 (5!) **3)** 10! **4)** $\frac{1}{2}(10!)$

18. There are two bags each containing m balls. If a man has to select equals number of balls from both the bags the number of ways in which he can do so if he must choose at least one ball from each bag is

1) m^2 **2)** ${}^{2m}C_m$ **3)** ${}^{2m}C_m - 1$ **4)** none of these

19. In an election, the number of candidates is one greater than the person to be elected. If a voter can vote in 254 ways, the number of candidates is

1) 7 **2)** 10 **3)** 8 **4)** 6

20. The number of triangle that can be formed with 10 points as vertices, n of them being collinear, is 110. Then n is

1) 3 **2)** 4 **3)** 5 **4)** 6

21. The number of ways to give 16 different things to three persons A,B,C so that B gets one more than A and C gets two more that B, is

1) $\frac{16!}{4!5!7!}$ **2)** $4!5!7!$ **3)** $\frac{16!}{3!5!8!}$ **4)** none of these

22. Let $x_1, x_2, ..., x_k$ be the divisors of positive integer 'n' (including 1 and n). If $x_1 + x_2 + ... + x_k = 75$, then $\sum_{i=1} 1/x_i$ is equal to

1) $\frac{75}{n^2}$ **2)** $\frac{75}{n}$ **3)** $\frac{75}{k}$ **4)** none of these

23. The total number of ways in which n^2 number of identical balls can be put in n numbered boxes (1,2,3,…..n) such that i^{th} box contains at least i number of balls is

1) ${}^{n^2}C_{n-1}$ **2)** ${}^{n^2-1}C_{n-1}$ **3)** ${}^{\frac{n^2+n-2}{2}}C_{n-1}$ **4)** none of these

24. Number of ways in which 25 identical things be distributed among five persons if each gets odd number of things is

1) ${}^{25}C_4$ **2)** ${}^{12}C_8$ **3)** ${}^{14}C_{10}$ **4)** ${}^{13}C_3$

25. The total number of ways in which three distinct numbers in A.P. can be selected from the set $\{1, 2, 3, ...24\}$ is equal to

1) 66 **2)** 132 **3)** 198 **4)** none of these

26. The number of three-digit numbers of the form xyz such that $x < y$ and $z \le y$ is

1) 276 **2)** 285 **3)** 240 **4)** 244

27. A man has three friends. The number of ways he can invite one friend everyday for dinner on six successive nights so that no friend is invited more than three times is

1) 642 **2)** 320 **3)** 420 **4)** 510

28. There are $(n+1)$ white and $(n+1)$ black balls each set numbered 1 to n +1. The number of ways in which the balls can be arranged in a row so that the adjacent balls are of different colours is

1) $(2n+2)!$ **2)** $(2n+2)!\times 2$ **3)** $(n+1)!\times 2$ **4)** $2\{(n+1)!\}^2$

29. The total number of positive integral solution of $15 < x_1 + x_2 + x_3 \le 20$ is equal to

1) 685 **2)** 785 **3)** 1125 **4)** none of these

30. If n objects are arranged in a row, then the number of ways of selecting three of these objects so that no two of them are next to each other is

1) ${}^{n-2}C_3$ **2)** ${}^{n-3}C_2$ **3)** ${}^{n-3}C_3$ **4)** none of these

KEY SHEET

1)	2	**2)**	4	**3)**	3	**4)**	1	**5)**	2	**6)**	2	**7)**	1
8)	3	**9)**	1	**10)**	1	**11)**	2	**12)**	3	**13)**	3	**14)**	4
15)	3	**16)**	4	**17)**	4	**18)**	2	**19)**	3	**20)**	3	**21)**	1
22)	2	**23)**	3	**24)**	3	**25)**	2	**26)**	1	**27)**	4	**28)**	4
29)	1	**30)**	1										